Positive corona plasma electric field, boundary radius and voltage drop (derivation and table of values)

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Air characteristics

******Relative air density d******

d = T_oP/TP_o , where T_o and P_o are standard temp 293 K and pressure 101325 Pa

-> Example: d=1 (standard T and P)

******Breakdown (ionizing) electric field in air Eo******

E_o = 3.0*d MV/m
proportional to air density
equivalent to a molecule density reduced field E/N of 120 Td where 1 Td = 10^{-21} V m^2

-> Example: E_o=3.0*1 MV/m = 3MV/m (30kV/cm)

Positive corona plasma characteristics

******Ratio of electric field E_i at wire surface to E_o, E_i/E_o******

E_i is Peek's empirical corona onset field for smooth wire. The assumption that the electric field at the wire surface equals Peek's onset field is generally accepted and justified by Morrow (1997), cf Chen [1].

$E_i / E_o = 1 + 0.03 / (d * r_w)^{1/2}$, where r_w is wire radius in m

-> Example: 0.2mm wire (r_w = 100µm)

$E_i / E_o = 1 + 0.03 / (1 * 100\mu)^{1/2} = 1 + 3 = 4$
E_i = 4 * 3 MV/m = 12 MV/m (120kV/cm)

******Electric field at radius r in plasma region E(r)******

(Laplacian since net space charge is negligible in plasma)

$E(r) = (E_i * r_w) / r$ (monotonically decreasing)

-> Example: E(r)=(12MV/m *100µm)/r = 1200V/r

See curve below, from Chen [1]

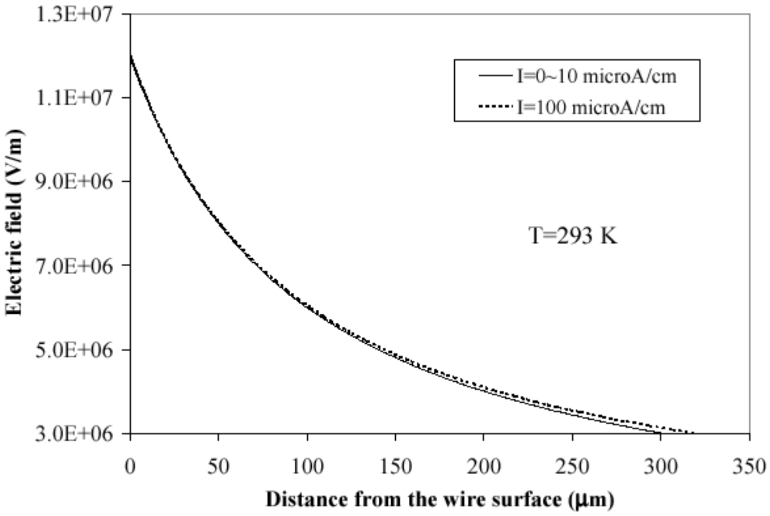

Fig. 2-8. Electric field distribution for a 100 µm radius wire. The air temperature is 293

****** External plasma boundary radius r_o ******

(where E gets below breakdown E_o, ie $E_i * r_w / r_o = E_o$)

$r_o = E_i / E_o * r_w$
this depends only on wire radius and air density, NOT on collector geometry or current cf curve.

-> Example: $r_o = 4*100\mu = 0.4mm$

******Voltage drop in plasma V_d******

Voltage drop V_d = Integral from r_w to r_o of E(r)dr

$= E_i * r_w *$ integral from r_w to r_o of (1/r)dr

$= E_i * r_w * (\mathrm{Ln}(r_o) - \mathrm{Ln}(r_w)))$

$= E_i * r_w * \mathrm{Ln}(r_o / r_w)$, replacing r_o by $E_i / E_o * r_w$, we get

$V_d = E_i * r_w * \mathrm{Ln}(E_i/E_o)$

-> Example: V_d = 12MV/m * 100µm * Ln(4)= 1200V * Ln(4)=1664 V

Note: Boundary conditions (r_o, E_o) and voltage drop V_d are all that matters for EHD calculations and simulations (no net space charge i.e. no thrust within plasma).

Table of values for various wire diameters d_w in std air:

d_w(mm)	r_w(mm)	E_i/E_o	r_o(mm)	V_d(V)
0.02	0.01	10.49	0.10	739
0.04	0.02	7.71	0.15	945
0.06	0.03	6.48	0.19	1089
0.08	0.04	5.74	0.23	1205
0.10	0.05	5.24	0.26	1303
0.12	0.06	4.87	0.29	1389
0.14	0.07	4.59	0.32	1467
0.16	0.08	4.35	0.35	1537
0.18	0.09	4.16	0.37	1603
0.20	0.10	4.00	0.40	1664

Note 1: In EHD thrusters, for a typical 0.1mm wire, power dissipation in plasma is low, less than 0.07W/cm (V_d * max current assumed to be 50µA/cm for half cylindrical design).

Since, moreover, ionic wind provides air-cooling, plasma T and P must be very close to ambient (in my opinion).

Note 2: For smaller gaps/voltages (a few mm, a few kV), voltage drop in plasma becomes a sizeable portion of total voltage so power dissipation in plasma cannot be neglected, especially for larger wires. For typical EHD thruster gaps/voltages (a few tens of mm, a few tens of kV) this is negligible.

References:

[1] Junhong Chen, PhD thesis, U. of Minnesota, 2002
http://www.menet.umn.edu/~jhchen/Junhong_dissertation_final.pdf