Cockroft-Walton Optimum Design Guide V2.0

Theoretical derivation

Cockroft-Walton output voltage as a function of number of stages n, peak input voltage Epk, output current Iload, and product fc of frequency f and capacitance c (cf e.g. http://www.blazelabs.com/e-exp15.asp) ; Note that as f and c only appear as their product in most equations, I have turned them into a single parameter fc. They will only be separated in the final part of the numerical application section.

```
Eout=2*n*Epk-Iload*(4*n^3+3*n^2-n)/(6*fc);
```

For later use let's find the expression for fc as a function of other parameters and wanted output voltage Vout

```
Solve[Eout==Vout,fc]
```

$$\left\{\left\{fc \to \frac{-Iload\, n + 3\, Iload\, n^2 + 4\, Iload\, n^3}{12\, Epk\, n - 6\, Vout}\right\}\right\}$$

$$fc[Nstages\_] = Iload\,\frac{-Nstages + 3\, Nstages^2 + 4\, Nstages^3}{12\, Epk\, Nstages - 6\, Vout};$$

Now back to the Eout expression. Due to the fast growth of the n^3 term in the negative term (voltage drop relative to the no-load value), if n starts from zero and increases without changing other parameters, Eout first increases, then reaches a peak, and then decreases. Let's derive the optimum number of stages i.e. the n for which Eout[n] is maximum. The derivative of Eout with respect to n is zero at peak Eout:

```
Solve[D[Eout,n]==0,n]
```

$$\left\{\left\{n \to \frac{-3\, Iload - \sqrt{3}\,\sqrt{48\, Epk\, fc\, Iload + 7\, Iload^2}}{12\, Iload}\right\},\ \left\{n \to \frac{-3\, Iload + \sqrt{3}\,\sqrt{48\, Epk\, fc\, Iload + 7\, Iload^2}}{12\, Iload}\right\}\right\}$$

The above positive solution for n, or rather an approximative formula derived by neglecting the 3*n^2-n term in the Eout expression before taking the derivative, was used up to now to find the optimum number of stages. The problem is, optimum n depends on fc, and fc depends on n, so engineers had to proceed by trial and error to find adequate values of fc and n yielding both the required output voltage and max Eout as a function of n.

To avoid this fastidious trial and error process lets' be bold and try and solve **simultaneously** the equations "Eout= wanted output voltage Vout" and "derivative of Eout with respect to n = zero" for fc and n. Note you can happily skip the lengthy solutions below to jump right to the optimum n formula.

```
Simplify[Solve[{Eout==Vout,D[Eout,n]==0},{fc,n}],{Epk>0,Vout>Epk,fc>0,n>=1}]
```

$$\{\{\text{fc} \to (\text{Iload} (-3\,\text{Epk}^4 + 3\,\text{Epk}^3 (-40\,\text{Vout} + (-\text{Epk}^3 - 22\,\text{Epk}^2\,\text{Vout} + 8\,\text{Vout}^3 + 4\,\text{Epk}$$

$$(3\,\text{Vout}^2 + \sqrt{\text{Vout}\,(2\,\text{Epk}^3 + 25\,\text{Epk}^2\,\text{Vout} - 44\,\text{Epk}\,\text{Vout}^2 - 28\,\text{Vout}^3)}))^{1/3}) +$$

$$\text{Epk}^2 (-120\,\text{Vout}^2 + 24\sqrt{\text{Vout}\,(2\,\text{Epk}^3 + 25\,\text{Epk}^2\,\text{Vout} - 44\,\text{Epk}\,\text{Vout}^2 - 28\,\text{Vout}^3)} -$$

$$30\,\text{Vout} (-\text{Epk}^3 - 22\,\text{Epk}^2\,\text{Vout} + 8\,\text{Vout}^3 +$$

$$4\,\text{Epk} (3\,\text{Vout}^2 + \sqrt{\text{Vout}\,(2\,\text{Epk}^3 + 25\,\text{Epk}^2\,\text{Vout} - 44\,\text{Epk}\,\text{Vout}^2 - 28\,\text{Vout}^3)}))^{1/3} -$$

$$19 (-\text{Epk}^3 - 22\,\text{Epk}^2\,\text{Vout} + 8\,\text{Vout}^3 + 4\,\text{Epk} (3\,\text{Vout}^2 +$$

$$\sqrt{\text{Vout}\,(2\,\text{Epk}^3 + 25\,\text{Epk}^2\,\text{Vout} - 44\,\text{Epk}\,\text{Vout}^2 - 28\,\text{Vout}^3)}))^{2/3}) +$$

$$36\,\text{Vout}^2 (4\,\text{Vout}^2 + 2\,\text{Vout} (-\text{Epk}^3 - 22\,\text{Epk}^2\,\text{Vout} + 8\,\text{Vout}^3 + 4\,\text{Epk}$$

$$(3\,\text{Vout}^2 + \sqrt{\text{Vout}\,(2\,\text{Epk}^3 + 25\,\text{Epk}^2\,\text{Vout} - 44\,\text{Epk}\,\text{Vout}^2 - 28\,\text{Vout}^3)}))^{1/3} +$$

$$(-\text{Epk}^3 - 22\,\text{Epk}^2\,\text{Vout} + 8\,\text{Vout}^3 + 4\,\text{Epk} (3\,\text{Vout}^2 +$$

$$\sqrt{\text{Vout}\,(2\,\text{Epk}^3 + 25\,\text{Epk}^2\,\text{Vout} - 44\,\text{Epk}\,\text{Vout}^2 - 28\,\text{Vout}^3)}))^{2/3}) +$$

$$12\,\text{Epk} (24\,\text{Vout}^3 + 4\,\text{Vout}\sqrt{\text{Vout}\,(2\,\text{Epk}^3 + 25\,\text{Epk}^2\,\text{Vout} - 44\,\text{Epk}\,\text{Vout}^2 - 28\,\text{Vout}^3)} +$$

$$9\,\text{Vout}^2 (-\text{Epk}^3 - 22\,\text{Epk}^2\,\text{Vout} + 8\,\text{Vout}^3 +$$

$$4\,\text{Epk} (3\,\text{Vout}^2 + \sqrt{\text{Vout}\,(2\,\text{Epk}^3 + 25\,\text{Epk}^2\,\text{Vout} - 44\,\text{Epk}\,\text{Vout}^2 - 28\,\text{Vout}^3)}))^{1/3} +$$

$$\sqrt{\text{Vout}\,(2\,\text{Epk}^3 + 25\,\text{Epk}^2\,\text{Vout} - 44\,\text{Epk}\,\text{Vout}^2 - 28\,\text{Vout}^3)}$$

$$(-\text{Epk}^3 - 22\,\text{Epk}^2\,\text{Vout} + 8\,\text{Vout}^3 +$$

$$4\,\text{Epk} (3\,\text{Vout}^2 + \sqrt{\text{Vout}\,(2\,\text{Epk}^3 + 25\,\text{Epk}^2\,\text{Vout} - 44\,\text{Epk}\,\text{Vout}^2 - 28\,\text{Vout}^3)}))^{1/3} +$$

$$3\,\text{Vout} (-\text{Epk}^3 - 22\,\text{Epk}^2\,\text{Vout} + 8\,\text{Vout}^3 + 4\,\text{Epk} (3\,\text{Vout}^2 +$$

$$\sqrt{\text{Vout}\,(2\,\text{Epk}^3 + 25\,\text{Epk}^2\,\text{Vout} - 44\,\text{Epk}\,\text{Vout}^2 - 28\,\text{Vout}^3)}))^{2/3}))) \Big/$$

$$(192\,\text{Epk}^3 (-\text{Epk}^3 - 22\,\text{Epk}^2\,\text{Vout} + 8\,\text{Vout}^3 + 4\,\text{Epk} (3\,\text{Vout}^2 +$$

$$\sqrt{\text{Vout}\,(2\,\text{Epk}^3 + 25\,\text{Epk}^2\,\text{Vout} - 44\,\text{Epk}\,\text{Vout}^2 - 28\,\text{Vout}^3)}))^{2/3}),$$

$$n \to \frac{1}{8\,\text{Epk}} (-\text{Epk} + 2\,\text{Vout} + (\text{Epk} + 2\,\text{Vout})^2 \Big/ (-\text{Epk}^3 - 22\,\text{Epk}^2\,\text{Vout} + 8\,\text{Vout}^3 +$$

$$4\,\text{Epk} (3\,\text{Vout}^2 + \sqrt{\text{Vout}\,(2\,\text{Epk}^3 + 25\,\text{Epk}^2\,\text{Vout} - 44\,\text{Epk}\,\text{Vout}^2 - 28\,\text{Vout}^3)}))^{1/3} +$$

$$(-\text{Epk}^3 - 22\,\text{Epk}^2\,\text{Vout} + 8\,\text{Vout}^3 + 4\,\text{Epk} (3\,\text{Vout}^2 +$$

$$\sqrt{\text{Vout}\,(2\,\text{Epk}^3 + 25\,\text{Epk}^2\,\text{Vout} - 44\,\text{Epk}\,\text{Vout}^2 - 28\,\text{Vout}^3)}))^{1/3})\},$$

$$\{\text{fc} \to \frac{1}{12\,\text{Epk}} (\text{Iload} (-1 + \frac{1}{27648\,\text{Epk}^2} ((72\,(\text{Epk} - 2\,\text{Vout}) + (36\,(1 + i\sqrt{3})$$

$$(\text{Epk} + 2\,\text{Vout})^2) \Big/ (-\text{Epk}^3 - 22\,\text{Epk}^2\,\text{Vout} + 8\,\text{Vout}^3 + 4\,\text{Epk}$$

$$(3\,\text{Vout}^2 + \sqrt{\text{Vout}\,(2\,\text{Epk}^3 + 25\,\text{Epk}^2\,\text{Vout} - 44\,\text{Epk}\,\text{Vout}^2 - 28\,\text{Vout}^3)}))^{1/3} -$$

$$36\,i\,(i + \sqrt{3}) (-\text{Epk}^3 - 22\,\text{Epk}^2\,\text{Vout} + 8\,\text{Vout}^3 + 4\,\text{Epk} (3\,\text{Vout}^2 +$$

$$\sqrt{Vout\ (2\ Epk^3 + 25\ Epk^2\ Vout - 44\ Epk\ Vout^2 - 28\ Vout^3)}\ ))^{1/3})^2) +$$

$$\frac{1}{96\ Epk}\ \Big(-72\ (Epk - 2\ Vout) - \big(36\ i\ (-i + \sqrt{3})\ (Epk + 2\ Vout)^2\big)\Big/$$

$$\Big(-Epk^3 - 22\ Epk^2\ Vout + 8\ Vout^3 +$$

$$4\ Epk\ \Big(3\ Vout^2 + \sqrt{Vout\ (2\ Epk^3 + 25\ Epk^2\ Vout - 44\ Epk\ Vout^2 - 28\ Vout^3)}\ \Big)\Big)^{1/3} +$$

$$36\ i\ (i + \sqrt{3})\ \Big(-Epk^3 - 22\ Epk^2\ Vout + 8\ Vout^3 + 4\ Epk\ \Big(3\ Vout^2 +$$

$$\sqrt{Vout\ (2\ Epk^3 + 25\ Epk^2\ Vout - 44\ Epk\ Vout^2 - 28\ Vout^3)}\ \Big)\Big)^{1/3}\Big)\Big)\Big),$$

$$n \to \frac{1}{576\ Epk}\ \Big(-72\ (Epk - 2\ Vout) - \big(36\ i\ (-i + \sqrt{3})\ (Epk + 2\ Vout)^2\big)\Big/$$

$$\Big(-Epk^3 - 22\ Epk^2\ Vout + 8\ Vout^3 +$$

$$4\ Epk\ \Big(3\ Vout^2 + \sqrt{Vout\ (2\ Epk^3 + 25\ Epk^2\ Vout - 44\ Epk\ Vout^2 - 28\ Vout^3)}\ \Big)\Big)^{1/3} +$$

$$36\ i\ (i + \sqrt{3})\ \Big(-Epk^3 - 22\ Epk^2\ Vout + 8\ Vout^3 + 4\ Epk$$

$$\Big(3\ Vout^2 + \sqrt{Vout\ (2\ Epk^3 + 25\ Epk^2\ Vout - 44\ Epk\ Vout^2 - 28\ Vout^3)}\ \Big)\Big)^{1/3}\Big)\Big\},$$

$$\Big\{fc \to \frac{1}{12\ Epk}\ \Big(Iload\ \Big(-1 + \frac{1}{96\ Epk}\ \Big(-72\ (Epk - 2\ Vout) + \big(36\ i\ (i + \sqrt{3})\ (Epk + 2\ Vout)^2\big)\Big/$$

$$\Big(-Epk^3 - 22\ Epk^2\ Vout + 8\ Vout^3 +$$

$$4\ Epk\ \Big(3\ Vout^2 + \sqrt{Vout\ (2\ Epk^3 + 25\ Epk^2\ Vout - 44\ Epk\ Vout^2 - 28\ Vout^3)}\ \Big)\Big)^{1/3} -$$

$$36\ (1 + i\ \sqrt{3})\ \Big(-Epk^3 - 22\ Epk^2\ Vout + 8\ Vout^3 + 4\ Epk\ \Big(3\ Vout^2 +$$

$$\sqrt{Vout\ (2\ Epk^3 + 25\ Epk^2\ Vout - 44\ Epk\ Vout^2 - 28\ Vout^3)}\ \Big)\Big)^{1/3}\Big) +$$

$$\frac{1}{27648\ Epk^2}\ \Big(\Big(72\ (Epk - 2\ Vout) - \big(36\ i\ (i + \sqrt{3})\ (Epk + 2\ Vout)^2\big)\Big/$$

$$\Big(-Epk^3 - 22\ Epk^2\ Vout + 8\ Vout^3 + 4\ Epk$$

$$\Big(3\ Vout^2 + \sqrt{Vout\ (2\ Epk^3 + 25\ Epk^2\ Vout - 44\ Epk\ Vout^2 - 28\ Vout^3)}\ \Big)\Big)^{1/3} +$$

$$36\ (1 + i\ \sqrt{3})\ \Big(-Epk^3 - 22\ Epk^2\ Vout + 8\ Vout^3 + 4\ Epk\ \Big(3\ Vout^2 +$$

$$\sqrt{Vout\ (2\ Epk^3 + 25\ Epk^2\ Vout - 44\ Epk\ Vout^2 - 28\ Vout^3)}\ \Big)\Big)^{1/3}\Big)^2\Big)\Big)\Big),$$

$$n \to \frac{1}{576\ Epk}\ \Big(-72\ (Epk - 2\ Vout) + \big(36\ i\ (i + \sqrt{3})\ (Epk + 2\ Vout)^2\big)\Big/$$

$$\Big(-Epk^3 - 22\ Epk^2\ Vout + 8\ Vout^3 +$$

$$4\ Epk\ \Big(3\ Vout^2 + \sqrt{Vout\ (2\ Epk^3 + 25\ Epk^2\ Vout - 44\ Epk\ Vout^2 - 28\ Vout^3)}\ \Big)\Big)^{1/3} -$$

$$36\ (1 + i\ \sqrt{3})\ \Big(-Epk^3 - 22\ Epk^2\ Vout + 8\ Vout^3 + 4\ Epk$$

$$\Big(3\ Vout^2 + \sqrt{Vout\ (2\ Epk^3 + 25\ Epk^2\ Vout - 44\ Epk\ Vout^2 - 28\ Vout^3)}\ \Big)\Big)^{1/3}\Big)\Big\}\Big\}$$

Now let's forget about the gruesome fc expressions which will be slightly wrong when we round n to the closest integer anyway, and lets' pick the relatively simple expression for n in the first solution pair (which I have found to be the only meaningful one in numerical trials) and simplify it (forgive the cooking details).

```
n =
 FullSimplify[1/(8 Epk) (-Epk + 2 Vout + (Epk + 2 Vout)^2 / (-Epk^3 - 22 Epk^2 Vout + 8 Vout^3 + 4 Epk
      (3 Vout^2 + Sqrt[Vout (2 Epk^3 + 25 Epk^2 Vout - 44 Epk Vout^2 - 28 Vout^3)]))^(1/3) +
   (-Epk^3 - 22 Epk^2 Vout + 8 Vout^3 + 4 Epk (3 Vout^2 +
      Sqrt[Vout (2 Epk^3 + 25 Epk^2 Vout - 44 Epk Vout^2 - 28 Vout^3)]))^(1/3))]
```

$$\frac{1}{8\,\text{Epk}}\left(-\text{Epk}+2\,\text{Vout}+(\text{Epk}+2\,\text{Vout})^2\Big/\left(-\text{Epk}^3-22\,\text{Epk}^2\,\text{Vout}+8\,\text{Vout}^3+4\,\text{Epk}\left(3\,\text{Vout}^2+\sqrt{(\text{Epk}-2\,\text{Vout})\,\text{Vout}\,(2\,\text{Epk}+\text{Vout})\,(\text{Epk}+14\,\text{Vout})}\right)\right)^{1/3}+\left(-\text{Epk}^3-22\,\text{Epk}^2\,\text{Vout}+8\,\text{Vout}^3+4\,\text{Epk}\left(3\,\text{Vout}^2+\sqrt{(\text{Epk}-2\,\text{Vout})\,\text{Vout}\,(2\,\text{Epk}+\text{Vout})\,(\text{Epk}+14\,\text{Vout})}\right)\right)^{1/3}\right)$$

```
n = Simplify[ComplexExpand[n], {Epk > 0, Vout > Epk}]
```

$$\frac{1}{8\,\text{Epk}}\left(-\text{Epk}+2\,\text{Vout}+2\,(\text{Epk}+2\,\text{Vout})\,\text{Cos}\left[\frac{1}{3}\,\text{Arg}\left[-\text{Epk}^3-22\,\text{Epk}^2\,\text{Vout}+8\,\text{Vout}^3+4\,\text{Epk}\left(3\,\text{Vout}^2+\sqrt{(\text{Epk}-2\,\text{Vout})\,\text{Vout}\,(2\,\text{Epk}+\text{Vout})\,(\text{Epk}+14\,\text{Vout})}\right)\right]\right]\right)$$

```
n = 1/(8 Epk)
  (-Epk + 2 Vout + 2 (Epk + 2 Vout) Cos[1/3 Arg[-Epk^3 - 22 Epk^2 Vout + 8 Vout^3 + 12 Epk Vout^2 +
       I * 4 Epk Sqrt[(2 Vout - Epk) Vout (2 Epk + Vout) (Epk + 14 Vout)]]]);
```

$$n=\frac{1}{8\,\text{Epk}}\left(-\text{Epk}+2\,\text{Vout}+2\,(\text{Epk}+2\,\text{Vout})\,\text{Cos}\left[\frac{1}{3}\,\text{ArcTan}\left[\frac{4\,\text{Epk}\sqrt{\text{Vout}\,(2\,\text{Epk}+\text{Vout})\,(-\text{Epk}+2\,\text{Vout})\,(\text{Epk}+14\,\text{Vout})}}{-\text{Epk}^3-22\,\text{Epk}^2\,\text{Vout}+12\,\text{Epk}\,\text{Vout}^2+8\,\text{Vout}^3}\right]\right]\right);$$

Note **optimum n depends on input and output voltages Epk and Vout only**, not on output current nor frequency or capacitance, this is a new result AFAIK and quite an improvement over previous expressions for Noptimum.

Furthermore, as Saviour Borg judiciously pointed out the day I posted v1.0 of this guide to the Blazelabs group, **optimum n in this expression depends only on the ratio of Vout to Epk** i.e. the voltage multiplying factor, which we will call x. This allowed me to further simplify the expression and post the essential results of the remainder of this theoretical section the next day to the group (post 2305 of 5 Aug 2005).

```
Nopt[x_]=FullSimplify[n/.Vout->x*Epk, {Epk>0, x>0}]
```

$$\frac{1}{8}\left(-1+2\,x+(2+4\,x)\,\text{Cos}\left[\frac{1}{3}\,\text{ArcTan}\left[\frac{4\sqrt{x\,(2+x)\,(-1+2\,x)\,(1+14\,x)}}{-1+2\,x\,(-11+6\,x+4\,x^2)}\right]\right]\right)$$

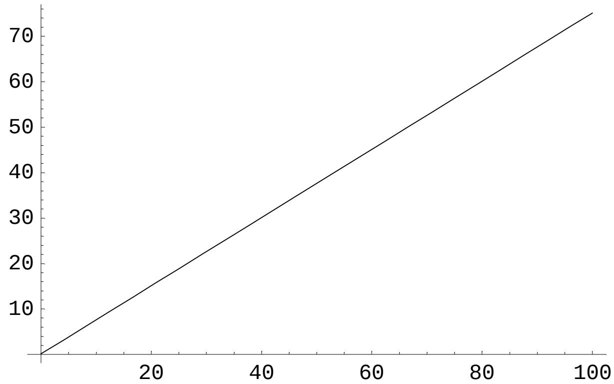
As an aside, let's put the exact Nopt[x] expression in a one-line form allowing copy-pasting the formula to e.g. an Excel spreadsheet

```
InputForm[Nopt[x]]
```

```
(-1 + 2*x + (2 + 4*x)*Cos[ArcTan[(4*Sqrt[x*(2 + x)*(-1 + 2*x)*(1 + 14*x)])/(-1 +
2*x*(-11 + 6*x + 4*x^2))]/3])/8
```

Now let's plot the function.

```
Plot[Nopt[x],{x,0,100}];
```



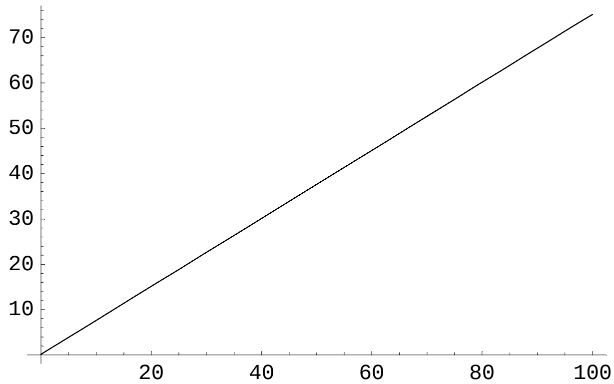
Looks surprisingly close to a straight line. Let's look for an asymptote. When x tends to infinity, the argument of ArcTan tends to 4x^2/8x^3 i.e. to 0, therefore so does the ArcTan, therefore the Cos tends to 1, i.e. the function's asymptote is $\frac{1}{8}\ (-1 + 2\ x + (2 + 4\ x)) = \frac{1}{8}\ (6\ x + 1)$ =3/4 x+1/8. Let's check:

```
Limit[Nopt[x]-(3/4 x+1/8), x->Infinity]
```

```
0
```

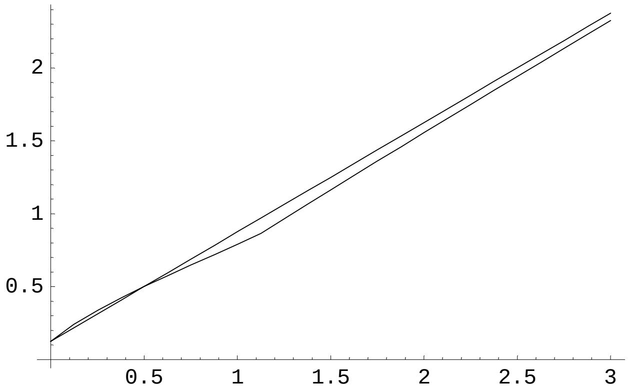
Ok, now let's plot the function **and** its asymptote for x = 0 to 100

```
Plot[{Nopt[x],3/4 x+1/8},{x,0,100}];
```



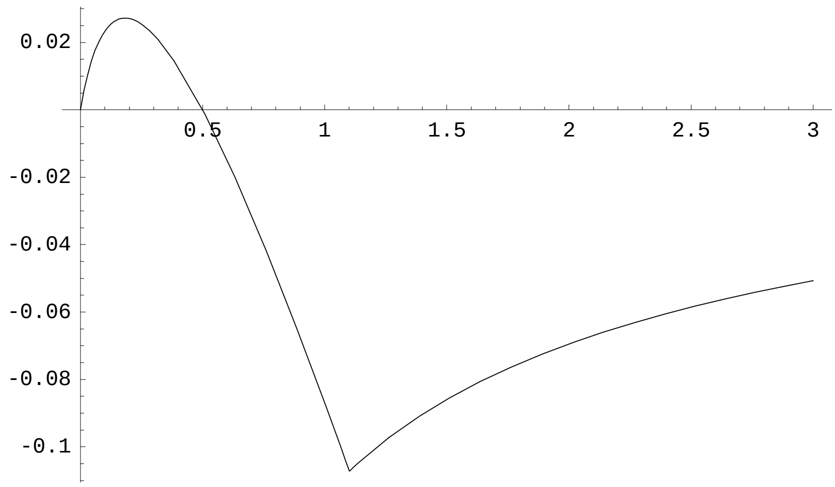
Looks very much like the previous plot: at this scale the function is obviously not distinguishable from its asymptote, let's zoom in on low x values

```
Plot[{Nopt[x],3/4 x+1/8},{x,0,3}];
```



They are still very close, let's plot the difference to find the bounds of the departure

```
Plot[Nopt[x]-(3/4 x+1/8),{x,0,3}];
```



We find that the function and its asymptote differ by less than +0.03/-0.11 from x=0 to infinity, and less than only +0/-0.05 from x=3 to infinity, which is negligible compared to the rounding "error" which can be as much as 0.5.

So we can gladly use the following close approximation

```
Noptapprox = 3/4 Vout/Epk + 1/8;
```

and round to the nearest integer.

I even encourage engineers to drop the 1/8 term in order to bias the rounding towards the smallest integer when the result is halfway between two integers. Being one at 0.5 below the Eout peak, and the other at 0.5 above, both numbers of stages would yield very nearly the same Eout value so the additional stage would be useless, and even detrimental as it would require a higher frequency and/or capacitor value for all stages (cf "How optimum is optimum?" section below).

```
Noptpractical=3/4 Vout/Epk;
```

We can conclude that n is optimum when the no-load value of Eout, 2*n*Epk, is approximately 2*(3/4 Vout/Epk)*Epk, i.e 1.5 times the desired value Vout, so one should simply aim the no-load value 2*n*Epk at 1.5 times the desired Vout, **even at low current** since optimum n doesn't depend on Iload! The fc product then follows from the expression derived in the beginning of this section, using optimum n as n. Then f can be chosen arbitrarily, and c can be deduced as fc/f, as illustrated in the numerical section below.

Numerical application

To demonstrate the above results let's specify Epk and Vout only, and deduce n from its expression

```
Epk=44.;Vout=1000.;n
```

```
17.1622
```

Let's check the approximated expressions yield very nearly the same result

```
Noptapprox
```

```
17.1705
```

```
Noptpractical
```

```
17.0455
```

Let's now round n to the nearest integer

```
n=Round[n]
```

```
17
```

Let's now specify output current Iload and deduce fc from the expression we found at the beginning

```
Iload = 0.01; fc[n]
```

```
0.0688911
```

Let's postulate that f is around 70kHz to find a first value for c

```
c=fc[n]/70000
```

9.84159×10^{-7}

Let's now adjust c to the nearest existing value

```
c=1 10^-6;
```

Now let's deduce the exact value for f

```
f=fc[n]/c
```

```
68891.1
```

Let's check that the ouput voltage value is really what we wanted.

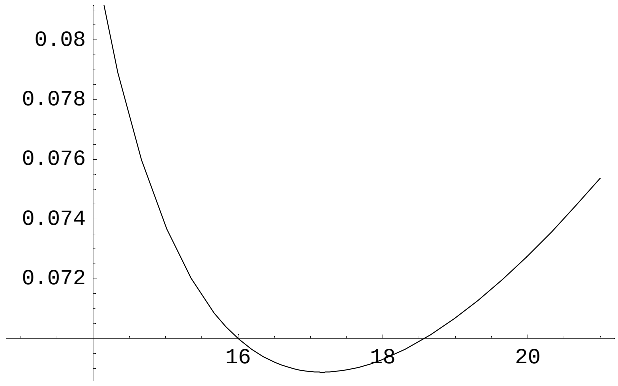
```
Eout_final=2*n*Epk-Iload*(4*n^3+3*n^2-n)/(6*f*c)
```

```
1000.
```

How optimum is optimum?

Let's see how the fc product would fare if we opted for less or more stages than the optimum n in the example above.

```
Plot[fc[n],{n,13,21}];
```



We see that the fc product is smallest at the optimum n value, i.e. for a given frequency the optimum n yields the smallest possible capacitance value required.

Let's now fix f at 70kHz

```
f=70000
```

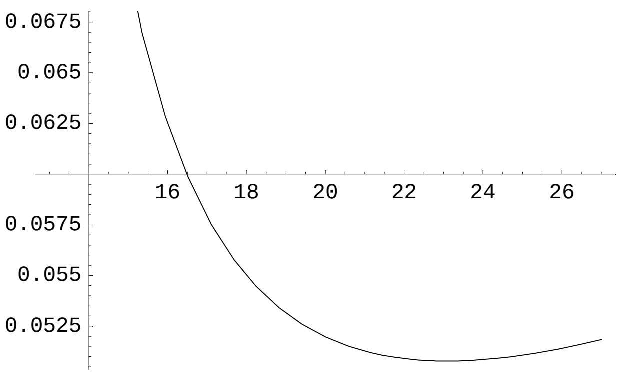
70000

and see how total energy stored in the capacitors evolves with n.
Average stage voltage is Vout/n
Average stored energy per capacitor is therefore 1/2 c (Vout/n)^2 roughly (to be exact it should be based on rms stage voltage for which I don't know the formula off hand)
Total stored energy in the 2*n capacitors is therefore $\frac{c\,Vout^2}{n} = \frac{fc\,Vout^2}{f\,n}$

$$Etot[nst\_] = \frac{fc[nst] * Vout^2}{f * nst}$$

$$\frac{0.142857\,(-nst + 3\,nst^2 + 4\,nst^3)}{nst\,(-6000. + 528.\,nst)}$$

```
Plot[Etot[n],{n,13,27}];
```



If the above formula for total stored energy is not too wrong, going for a lower number of stages than the optimum number calculated 17 would actually mean a higher total stored energy, which would be bad for security. On the contrary going for a higher number of stages (up to 23) would seem to yield a still lower total stored energy, but I wouldn't be surprised if the exact formula based on rms rather than average stage voltage reached its minimum at 17 stages rather than at 23 as does the approximate formula here, maybe someone will volunteer to derive the exact formula some day so we can clear this out.

All in all, our optimum number of stages seems, well... optimum.
